Tutorial on Particle Swarm Optimization

Jim Kennedy
Russ Eberhart

Jim Kennedy
Bureau of Labor Statistics
U. S. Department of Labor
Washington, DC
kennedy_jim@bls.gov
Particle Swarms
Part 1: Sociocognitive Optimization

Artificial Intelligence
Attempted to elicit intelligence from a computing machine by simulating human thought - good idea!

Early AI derived in the Dark Ages of psychology, when study of mind was taboo in science. Based on naive introspectionism.

Computational Intelligence
Perhaps we can apply more scientific concepts of mind.
• Self-report gives an unsatisfactory account of "real" cognition
• Sociocognition: thought as a social act
• Self-organization of societies, cultures
Sociocognition

Spanos
Levine, Resnick, and Higgins
Lewin's topological space

\[ B = f(P, E) \]
\[ (P, E) = LS \]

Cognitive Dissonance

Hypothesis 1: There are two major sources of cognition, namely, own experience and communication from others.


- Types of cognitive relations
- Minimization of dissonance
- Fundamental Attribution Error
- Exploration/exploitation

Note: vector=point
Latané’s Social Impact Theory

\[ i = f(SIN); \quad i = sN^t, \quad t < 1 \]

Latané (1981)

Nowak, Szamrej, and Latané
Psychological Review, 1990

Complex Adaptive Systems

Complexity (NK landscapes)

Self-organization
Emergence
(Immergence – downward causation)

Cellular automata
- Fixed point attractors
- Periodic attractors
- Chaotic attractors
- The "edge of chaos"

The Game of Life
Alife
Flocks, Herds, and Schools

- Heppner & Grenander
- Craig Reynolds

- Steer toward the center
- Match neighbors' velocity
- Avoid collisions
- (Seek roost)

Evolutionary Computation

Emulates natural evolution to "breed" solutions to hard problems, write computer programs, build robots, etc.

Population of proposed problem solutions

- Variation operators
- Mutation
- Crossover
- Selection

D.T. Campbell: "Blind variation and selective retention"
Particle Swarms

Sociocognitive space

- Certainly high-dimensional (i.e., Burgess & Lund)
- Abstract - attitudes, behaviors, cognition
- Heterogeneous with respect to evaluation (dissonance)
- Multiple individuals

Individual has position="mental state": $\vec{x}_i$
Individual changes: $\vec{v}_i$

Particle Swarms

Individuals learn from their own experience

$$\begin{align*}
\vec{v}_i &\leftarrow \vec{v}_i + \mathcal{U}(0, \varphi)(\vec{p}_i - \vec{x}_i) \\
\vec{x}_i &\leftarrow \vec{x}_i + \vec{v}_i
\end{align*}$$

This formula, iterated over time, causes each individual $i$’s trajectory to oscillate around its previous best point $p_i$ in the sociocognitive space.

It stochastically adjusts $i$’s velocity depending on previous successes, and occasionally updates $p_i$ - the previous best.
Oscillating trajectories (nonstochastic)

\[
\begin{align*}
v_i &\leftarrow v_i + (p_i - x_i) \\
x_i &\leftarrow x_i + v_i
\end{align*}
\]

- Oscillating Trajectories

Particle Swarms

Sociocognitive space can contain many individuals. They influence one another.

\[
\begin{align*}
v_i &\leftarrow v_i + U(0, \frac{\Omega}{2}) \otimes (p_i - x_i) + U(0, \frac{\Omega}{2}) \otimes (g - x_i) \\
x_i &\leftarrow x_i + v_i
\end{align*}
\]

*(g is neighborhood best)*

Evaluate your present position
Compare it to your previous best and neighborhood best
Imitate self and others
The “Drunkard’s Walk”

The particle will explode out of control if it is not limited in some way. Three methods have been widely used:

**Vmax**

\[
V_{\text{max}} = V_i + U(0, \frac{\Phi}{2}) \otimes (p_{\text{ai}} - x_i) + U(0, \frac{\Phi}{2}) \otimes (p_{\text{gi}} - x_i)
\]

if \( V_{\text{max}} > V \) max then \( V_i = V \) max;

else if \( V_{\text{max}} < -V \) max then \( V_i = -V \) max

**Inertia weight**

\[
V_i = \alpha V_i + U(0, C) \otimes (p_{\text{ai}} - x_i) + U(0, C) \otimes (p_{\text{gi}} - x_i)
\]

**Constriction coefficient**

\[
V_i = \chi(V_i + U(0, \frac{\Phi}{2}) \otimes (p_{\text{ai}} - x_i) + U(0, \frac{\Phi}{2}) \otimes (p_{\text{gi}} - x_i))
\]

(note that these last two are equivalent)

Binary Particle Swarms

\[
\begin{align*}
\vec{V}_i &\leftarrow \vec{V}_i + U(0, \frac{\Phi}{2}) \otimes (\vec{p}_i - \vec{x}_i) + U(0, \frac{\Phi}{2}) \otimes (\vec{p}_g - \vec{x}_i) \\
\text{if } U(0,1) < s(\vec{V}_i) &\text{ then } \vec{x}_i = 1 \\
\text{else } \vec{x}_i &\leftarrow 0
\end{align*}
\]

(Usu. Vmax, too)

where \( s(v) = \frac{1}{1+\exp(-v)} \)

logistic function

keeps it in (0,1)

Kennedy and Eberhart (1997)
Kennedy and Spears (1998)

Being looked at closely, expanded
Topology

Traditional ones: gbest, lbest

Topology determines how solutions spread through the population.

Gbest: immediate
Lbest: slowed

Affects the rate of convergence, parallelism of the search

Innovative topologies

Lots of variables to work with:
- Mean degree
- Clustering
- Heterogeneity
**FIPS**

"Fully Informed Particle Swarm" (Rui Mendes)

Should become the new standard

Distributes total $\phi$ across $n$ terms

$$
\bar{v}_i \leftarrow \chi\left(\bar{v}_i + \sum_{n=1}^{N_i} \frac{U(0,\phi) \otimes (\bar{p}_{nbr(n)} - \bar{x}_i)}{N_i}\right)
$$

$$
\bar{x}_i \leftarrow \bar{x}_i + \bar{v}_i
$$

Best neighbor is not selected
Individual not included in neighborhood
Dependent on topology

**FIPS Results**

Two performance metrics

Red: Topologies with average degree in the interval (4, 4.25).
Green: Topologies with average degree in the interval (3, 3.25) and clustering coefficient in the interval (0.1, 0.6).
Blue: Topologies with average degree in the interval (3, 3.25) and clustering coefficient in the interval (0.7, 0.9).
Light Blue: Topologies with average degree in the interval (5, 6) and clustering coefficient in the interval (0.025, 0.4).
Black: All other topologies.
Evolutionary Computation and Particle Swarms

Culture as evolution (anthropology)
Adaptation / learning
Memetics
Evolutionary epistemology

Change vs. selection
Fitness and dissonance
Cooperation vs. competition

"Cultures"

Kosko’s "South African" FCM
The Future of Particle Swarms

Trajectory Analysis

Includes constriction, inertia, convergence, etc.
Interaction Analysis

Individual trajectories very weak. Optimization is a function of interparticle interactions. The swarm as a whole, and as an aggregation of subpopulations. Effect on trajectory when new “bests” are found. “Immergence” and the effect of culture.

Probability Distribution Analysis

Theory: PS’s place among the EAs. Practice: New versions.
Parameters, Conditions, & Tweaks

- Initialization methods
- Population size
- Population diameter
- Absolute vs. signed velocities
- Population topology
- Births, deaths, migration
- Limiting domain (XMAX, VMAX)
- Multiobjective optimization
- “Subvector” techniques
- Comparison over problem spaces
- Hybrids

Particle Swarms As a General Problem-Solving Methodology

- Out of the computer...
- Replace eval() with human
- Replace particles with humans
- Networked computers (with or without users)
- Scientific research strategy
- Management & innovation
Sociocognition

Particle swarms as social-psychological theory
Not just “what,” but “why”
Social behavior → improved knowledge → adaptation
PS as a “simulation”